

# Complexity, Agent Based Models and Critical Mathematical Economics

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# Outline

## Morning:

- Simulations with NetLogo (GameofLife.netlogo)
- Introduction to Programming in “Java” with “Eclipse”
- Example: GameofLife.java

## Afternoon:

- Object-Oriented Programming
- Java Macro Agent-Based (JMAB) toolkit
- Critical Mathematical Economics and the Model-Theoretic Foundations of Controversies in Economic Policy

# The financial crisis and the crisis in economics

- Steve Keen: Debunking Economics (2002)
- The Financial Crisis and the Systemic Failure of Academic Economics, Dahlem Report 2008 (e.g. by Hans Foellmer)
- Adair Turner: Economics after the Crisis (2012)
- Institute for New Economic Thinking (INET, founded 2009)

# Mathematics for New Economic Thinking

- MNET: Fields Institute, 2013 (organized by M. Grasselli)
- Critical Mathematical Economics: Mathematics has been partly misused in mainstream economics to justify 'unregulated markets' before the crisis
- New approaches: e.g. Systemic Risk in Banking Systems, Revival of Keynes and Minsky in Macroeconomics

# Critical Mathematical Economics?

- Part A: (Mathematical) Critique of Mainstream Models (e.g. DSGE, SMD Theorems)
- Part B: Improve existing (heterodox) models with ingredients from Complexity (e.g. Dynamical Systems, ABM, Statistical Physics)
- Part C: Develop models for Policy Advice in new areas (e.g. Gender Aware Economics, Economic Democracy)

# Dynamic Stochastic General Equilibrium (DSGE)

- Standard model in Macroeconomics
- Seeks to explain the aggregate economy using theories based on strong microeconomic foundations.
- Collective decisions of rational individuals over a range of variables for both present and future.
- All variables are assumed to be simultaneously in equilibrium.

# DSGE and 'General Equilibrium Theory' as a Microfoundation

Theorem (from 'The theory of general economic equilibrium: A differentiable approach'. A. Mas-Colell, 1985).

Let  $z : S \rightarrow \mathbb{R}^l$  be a  $C^3$ - vector field satisfying some boundary conditions. Then for any  $\epsilon > 0$  we can find an economy  $E$  such that the excess demand function of  $E$  coincides with  $f$  on  $S_\epsilon$  and  $p$  is an equilibrium for  $E$  if and only if  $z(p) = 0$

It is also known as 'Sonnenschein-Mantel-Debreu Theorem', or 'anything goes' Theorem.

# SMD theorem: something is rotten in GE land





# Alternative Microfoundations: ABMs

- 'Agent Based Models' as a Microfoundation
- simulate the behaviour of heterogenous agents on the computer numerically
- Reference: e.g. 'Handbook of Computational Economics' (Volumes 1-3, most recent 2014)
- sometimes criticized as 'black box', as until recently no analytical solutions had been available
- Idea: Use methods from statistical physics to provide those by a mean-field approximation (see e.g. S. Landini, 2014)

# A probabilistic Micro-/Meso-foundation of Macroeconomics

‘The point is that precise behavior of each agent is irrelevant. Rather we need to recognize that microeconomic behavior is fundamentally stochastic.’ (Aoki and Yoshikawa 2006)

Idea:

- identifying homogeneous sub-populations of agents
- computing the mean-field variables (mean-field approximation), on an intermediate ‘Meso-level’
- master equation: study of the dynamics of the number of agents in each cluster, i.e. the transition probabilities
- Project with M.Grasselli and Patrick Li: Link this to Stock-Flow-Consistent Macroeconomic Models

# Back to Macroeconomics: Dynamic Stochastic General Equilibrium (DSGE)

- Equilibrium is only disrupted by exogenous shocks.
- The only way the economy can be in disequilibrium at any point in time is through decisions based on wrong information
- Money is neutral in its effect on real variables.
- Neoclassical economics is based on barter paradigm: money is convenient to eliminate the double coincidence of wants.  
DSGE is 'Macroeconomics without banks'!

# Minsky's alternative interpretation of Keynes

- In a modern economy, financial institutions determine the way funds are available for ownership of capital and production.
- Economy is fundamentally cyclical, with each state (boom, crisis, deflation, stagnation, expansion and recovery) containing the elements leading to the next in an identifiable manner.
- “Stability - or tranquility -is destabilizing” (in a world with a cyclical past and capitalist financial institutions).

# Minsky's Financial Instability Hypothesis

- Start when the economy is doing well but firms and banks are conservative. Most projects succeed -it pays to lever.
- Beginning of “euphoric economy”: increased debt to equity ratios, development of Ponzi/Speculative financier.
- Viability of business activity is eventually compromised.
- Ponzi financiers have to sell assets, liquidity dries out, asset market is flooded. Euphoria becomes a panic.

# Stock-Flow Consistent models

- Stock-flow consistent models emerged in the last decade as a common language for many heterodox schools of thought in economics.
- They consider both real and monetary factors simultaneously.
- Specify the balance sheet and transactions between sectors.
- Reject the RARE individual (representative agent with rational expectations) in favour of SAFE (sectoral average with flexible expectations) modelling.
- See Godley and Lavoie (2007) for the full framework.

# Goodwin Model - SFC matrix

Balance Sheet	Households	Firms		Sum
		current	capital	
Capital			$+pK$	$pK$
Sum (net worth)	0	0	$V_f$	$pK$
<b>Transactions</b>				
Consumption	$-pC$	$+pC$		0
Investment		$+pI$	$-pI$	0
Acct memo [GDP]		$[pY]$		
Wages	$+W$	$-W$		0
Profits		$-\Pi$	$+\Pi_u$	0
Sum	0	0	0	0
<b>Flow of Funds</b>				
Capital			$+pI$	$pI$
Sum	0	0	$\Pi_u$	$pI$
Change in Net Worth	0	$pI + \dot{p}K - p\delta K$		$\dot{p}K + p\dot{K}$

**Table:** SFC table for the Goodwin model.

# Goodwin Model (1967) - Assumptions

## ■ Assume that

$$N(t) = N_0 e^{\beta t} \quad (\text{total labour force})$$

$$a(t) = a_0 e^{\alpha t} \quad (\text{productivity per worker})$$

$$Y(t) = \nu K(t) = a(t)L(t) \quad (\text{total yearly output})$$

where  $K$  is the total stock of capital and  $L$  is the employed population.

## ■ Assume further that

$$\dot{w} = \Phi(\lambda)w \quad (\text{Phillips curve})$$

$$\dot{K} = (Y - wL) - \delta K \quad (\text{Say's Law})$$



# Goodwin Model - Differential equations

- Define

$$\omega = \frac{wL}{Y} = \frac{w}{a} \quad (\text{wage share})$$

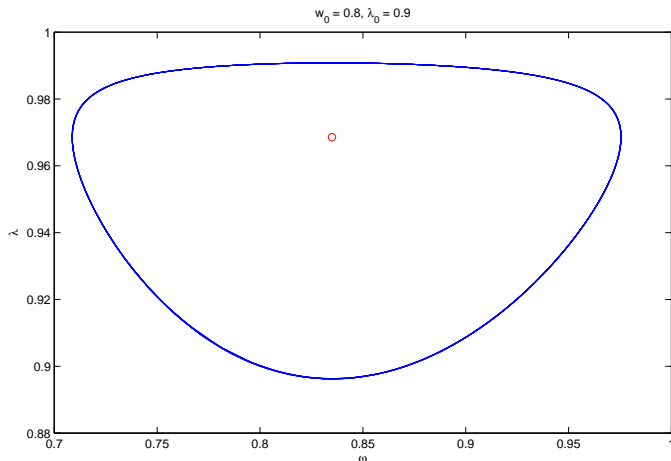
$$\lambda = \frac{L}{N} = \frac{Y}{aN} \quad (\text{employment rate})$$

- It then follows that

$$\dot{\omega} = \omega(\Phi(\lambda) - \alpha)$$

$$\dot{\lambda} = \lambda \left( \frac{1-\omega}{\nu} - \alpha - \beta - \delta \right)$$

# Example: Goodwin model



# SFC table for Keen (1995) model

Balance Sheet	Households	Firms		Banks	Sum
		current	capital		
Deposits	$+D$			$-D$	0
Loans			$-L$	$+L$	0
Capital			$+pK$		$pK$
Sum (net worth)	$V_h$	0	$V_f$	0	$pK$
<b>Transactions</b>					
Consumption	$-pC$	$+pC$			0
Investment		$+pl$	$-pl$		0
Acct memo [GDP]		$[pY]$			
Wages	$+W$	$-W$			0
Interest on deposits	$+rD$			$-rD$	0
Interest on loans		$-rL$		$+rL$	0
Profits		$-\Pi$	$+\Pi_u$		0
Sum	$S_h$	0	$S_f - pl$	0	0
<b>Flow of Funds</b>					
Deposits	$+\dot{D}$			$-\dot{D}$	0
Loans			$-\dot{L}$	$+\dot{L}$	0
Capital			$+pl$		$pl$
Sum	$S_h$	0	$\Pi_u$	0	$pl$
Change in Net Worth	$S_h$	$(S_f + pK - p\delta K)$			$pK + p\dot{K}$

Table: SFC table for the Keen model.

# Keen model - allow Investment through debt

- Assume now that new investment is given by

$$\dot{K} = \kappa(\pi)Y - \delta K$$

where  $\kappa(\cdot)$  is a nonlinear increasing function of profits  
 $\pi = 1 - \omega - rd$ .

- This leads to external financing through debt evolving according to

$$\dot{D} = \kappa(\pi)Y - \pi Y$$

# Keen model - Differential Equations

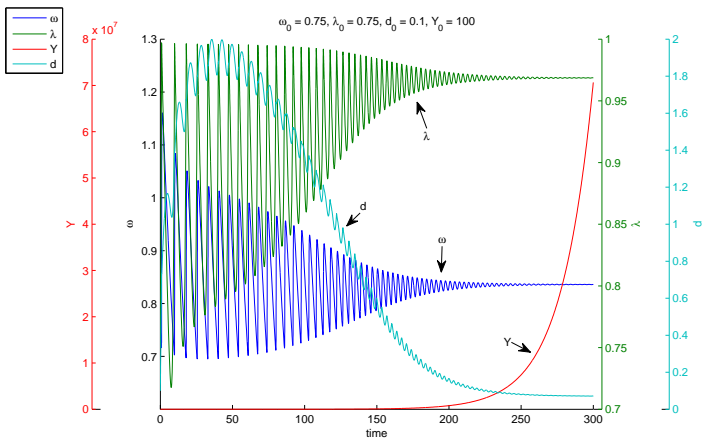
Denote the debt ratio in the economy by  $d = D/Y$ , the model can now be described by the following system

$$\dot{\omega} = \omega [\Phi(\lambda) - \alpha]$$

$$\dot{\lambda} = \lambda [g(\pi) - \alpha - \beta]$$

$$\dot{d} = \kappa(\pi) - \pi - dg(\pi)$$

# Example: convergence to the good equilibrium in a Keen model



# Example: explosive debt in a Keen model

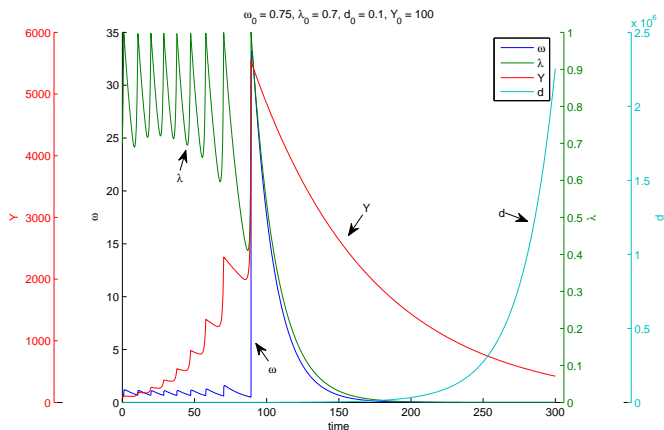
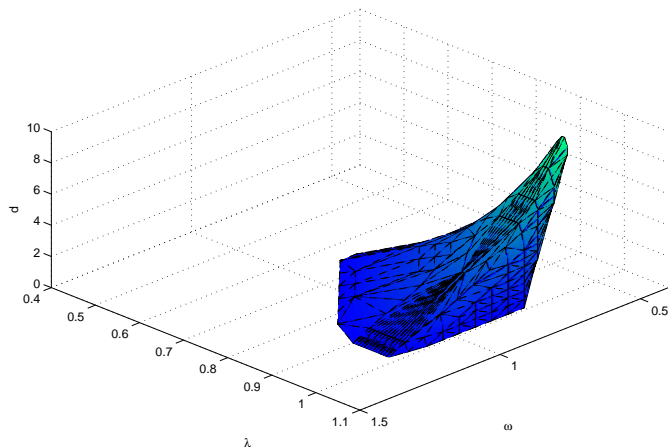


Figure: Grasselli and Costa Lima (2012)

# Basin of convergence for Keen model





# Ponzi financing

To introduce the destabilizing effect of purely speculative investment, we consider a modified version of the previous model with

$$\begin{aligned}\dot{D} &= \kappa(1 - \omega - rd)Y - (1 - \omega - rd)Y + P \\ \dot{P} &= \Psi(g(\omega, d)P\end{aligned}$$

where  $P$  stands for Ponzi (speculative) financing and  $\Psi(\cdot)$  is an increasing function of the growth rate of economic output.

# Example: Destabilizing effect of Ponzi financing

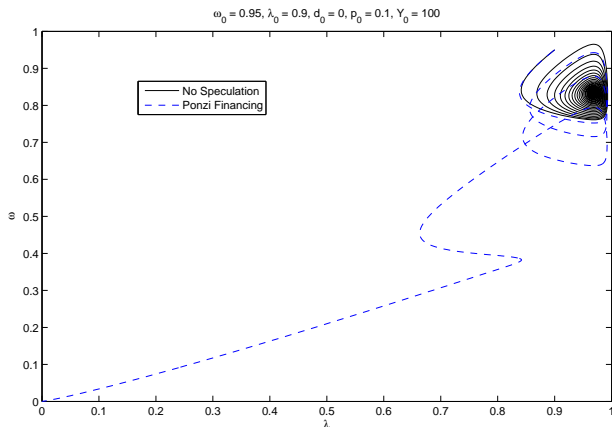


Figure: Grasselli and Costa Lima (2012)

# The Great Moderation in the U.S. - 1984 to 2007

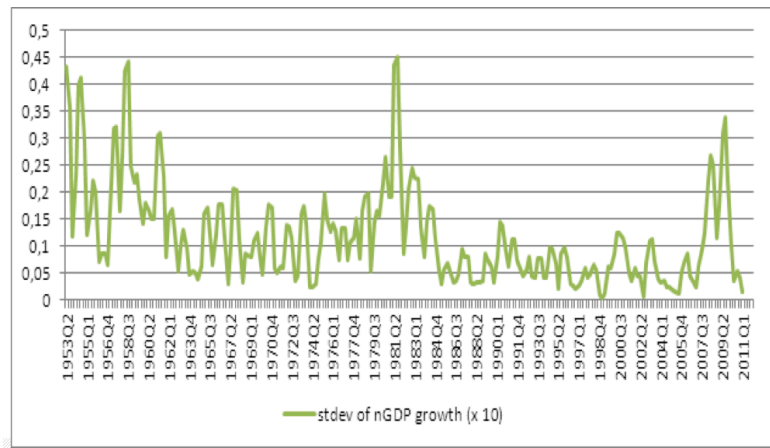


Figure: Grydaki and Bezemer (2013)

# Possible explanations

- Real-sector causes: inventory management, labour market changes, etc.
- Financial-sector causes: financial innovation, deregulation, better monetary policy, etc.
- Grydaki and Bezemer (2013): growth of debt in the real sector.

# Bank credit-to-GDP ratio in the U.S

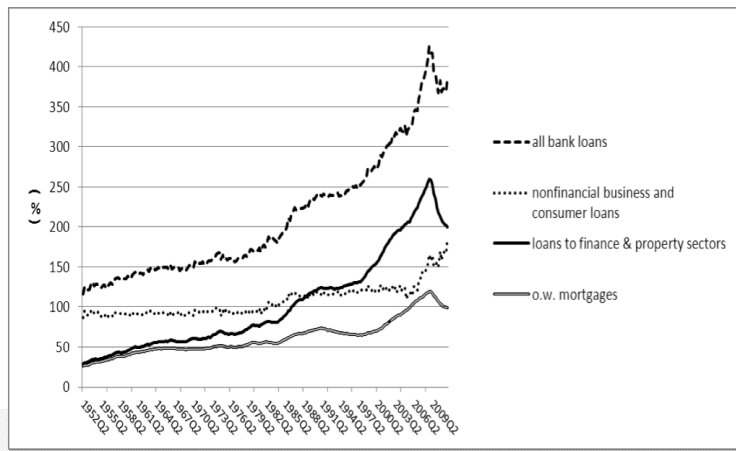
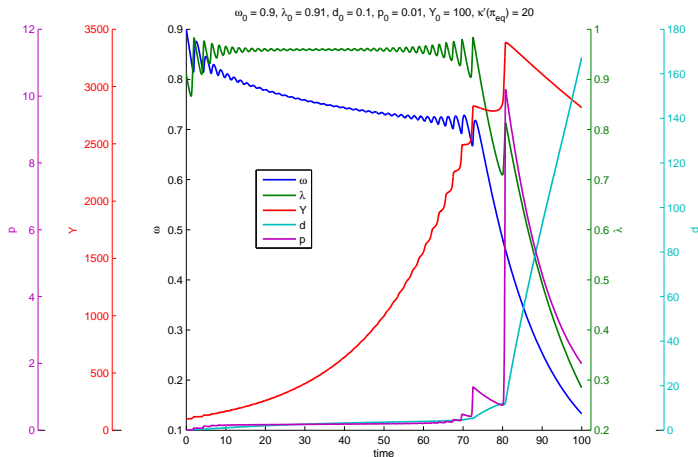


Figure: Grydaki and Bezemer (2013)

# Modelling the Great Moderation: Keen model with Ponzi financing

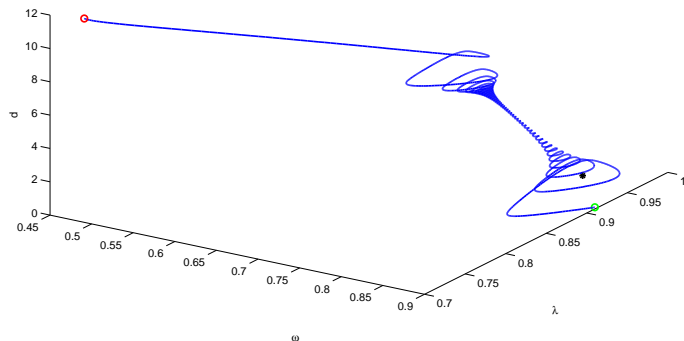
- Story: high debt  $\rightarrow$  growth with low volatility  $\rightarrow$  even higher debt  $\rightarrow$  explosive volatility
- Idea: choose initial conditions that are close to the basin of attraction of the equilibrium point with finite debt, but not inside it
- Result: long period of very low volatility, followed by explosion and collapse

# Example: strongly moderated oscillations



# Example (cont): Shilnikov bifurcation

$$\omega_0 = 0.9, \lambda_0 = 0.91, d_0 = 0.1, p_0 = 0.01, Y_0 = 100, \kappa'(\pi_{eq}) = 20$$





# Pseudo-Cycles and wage-led vs. profit-led

- Goodwin / Keen models: 'profit-led' or 'investment-driven' (investment is modelled, depends positively on profits, and consumption adjusts)
- Debt-Dynamics model: 'wage-led' or 'consumption-driven' (consumption is modelled, depends positively on wages, and investment adjusts)
- Research Question: Could there be a model that 'looks like' profit-led (e.g. by showing Goodwin-Cycles), but it is indeed wage-led?
- Joint with Charlie Brummit (Columbia), Torsten Heinrich (Bremen), Engelbert Stockhammer (Kingston)

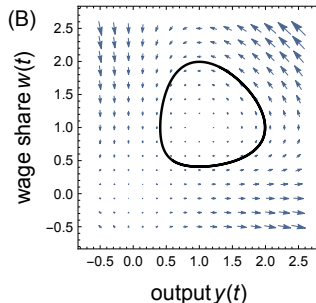
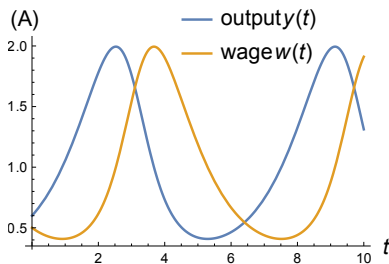
# A simplified 'toy' Goodwin Model

We follow [Stockhammer, Mitchel 2014]. They consider the following 'toy' Goodwin model

$$\begin{aligned}\dot{y} &= y(1 - w) \\ \dot{w} &= w(-c + ry)\end{aligned}$$

Note: This corresponds to the Goodwin model discussed above with parameters  $\alpha = \beta = \delta = 0$  and  $\nu = 1$ , where we have chosen a linear Phillips curve  $\Phi(\lambda) = -c + r\lambda$

## Example: 'Goodwin Cycles'



**Figure:** Sample orbit of the toy Goodwin model for  $r = c = 1$  and initial condition  $y(0) = 0.6$ ,  $w(0) = 0.5$ .

## Recall: Minsky's FIH - a Minsky Cycle

- Start when the economy is doing well but firms and banks are conservative. Most projects succeed -it pays to lever.
- Beginning of “euphoric economy”: increased debt to equity ratios, development of Ponzi/Speculative financier.
- Viability of business activity is eventually compromised.
- Ponzi financiers have to sell assets, liquidity dries out, asset market is flooded. Euphoria becomes a panic.

# A toy 2d Minsky Model

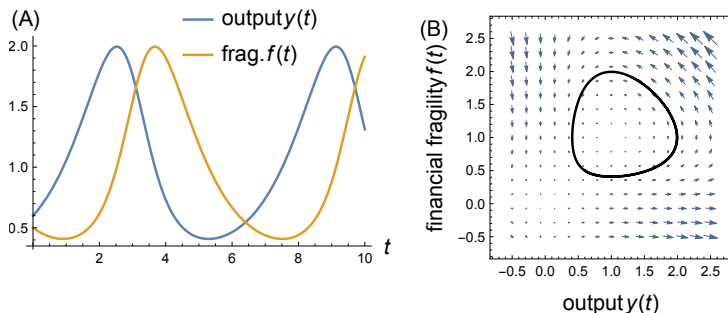
Let's consider the following 'toy Minsky Model' model

$$\dot{y} = y(1 - f) \quad (1)$$

$$\dot{f} = f(-1 + py) \quad (2)$$

Note:  $f$  stands for 'financial fragility' (think debt), and plays the role of the wage-share in the Goodwin model (the model above is equivalent to the toy Goodwin model for  $c=1$ ,  $w=f$ , and renaming  $r$  to  $p$ )

# Example: 'Minsky Cycles'



**Figure:** Sample orbit of the 2d Minsky model showing a 'Minsky Cycle' (for  $p = 1$  and initial condition  $y(0) = 0.6$ ,  $f(0) = 0.5$ ).

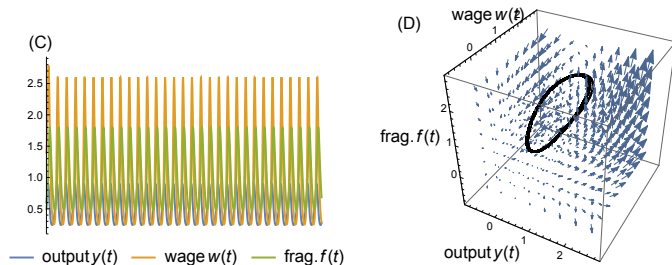
## A 3d Minsky Model with wage share

Finally, consider the following 3d Minsky Model, by adding a wage share variable (3) to equations (1) and (2):

$$\begin{aligned}\dot{y} &= y(1 - f) \\ \dot{f} &= f(-1 + py) \\ \dot{w} &= w(-c + ry - w)\end{aligned}\tag{3}$$

A *pseudo-Goodwin cycle* is a counterclockwise closed orbit in  $(y(t), w(t))$  such that  $y$  and  $w$  form a master-slave system (i.e. there is a feedback from  $y$  to  $w$ , but not vice-versa).

# Example: 'Pseudo Goodwin Cycles'



**Figure:** Sample orbit in 3d Minsky model: 'Pseudo Goodwin Cycle' (for  $p = 2, r = 5, c = 3/2$  and  $y(0) = 0.6, f(0) = 0.5, w(0) = 0.4$ ).



# General Pseudo-Cycles

- Goal 1: Try to find Pseudo Goodwin Cycles in a wage-led Minsky model and make it rigorous
- Goal 2: Generalize the 'Pseudo-Cycles' idea and relate to discussion on observational equivalence
- Note: Discussion if economy is profit-led' or 'wage-led' is very relevant for economic policy!

# Concluding remarks

- We discussed stock-flow macro-models and their micro-/meso-foundations.
- Applications included modelling the Great Moderation and the identification of Pseudo-Cycles
- The modelling framework is an alternative to the dominant microfounded DSGE paradigm in macroeconomics.
- Our approach opens up new avenues for the application of modern mathematical techniques to economics

# Promising Areas of Cooperation

- Hysteresis - Post-Keynesian Labour Market
- Networks Systemic Risk (e.g. Book by Tom Hurd)
- Mean Field Games and applications in economics
- Maybe even Turing instability, Delayed Feedback Control ...

New collaborators welcome - please check my homepage for updates (hint: google Critical Mathematical Economics :)

# Outlook

- Complexity and the Art of Public Policy (Colander, 2014):
- 'Complexity science is changing the way we think about social systems and social theory'
- Necessary to combine advanced technical methods with a critical view towards economic theory, largely unexplored field
- W. Elsner (2015): Policy Implications of Economic Complexity and Complexity Economics

# Example: Workshop Complexity Economics at Fields 2016

- took place at April 26-28, 2016 at Fields Institute Toronto, Canada ([www.fields.utoronto.ca/activities/15-16/complexity](http://www.fields.utoronto.ca/activities/15-16/complexity))
- organized by me in collaboration with Johannes Tiemer (Uni Duisburg-Essen)
- Talks by D. Colander (Middlebury College, USA) and I. Nikolic (TU Delft, Netherlands) are online!
- supported by the Young Scholars Initiative of INET

# Interesting Activities 2017

## Summer School on Macroeconomics Economic Policies

- 30.7.-5.8.2017 in Berlin, organized by IMK - Macroeconomic Policy Institute
- for graduate students (MA and PhD) and junior researchers, application deadline: 15th of March
- overview lectures on stock-flow-consistent and agent-based modelling

## EAEPE Annual Conference

- 19.-21.10. 2017 in Budapest, organized EAEPE - European Association for Evolutionary Political Economy
- introductory lecture for graduate students and junior researchers
- Special Session on “Agent-based macro models and post-Keynesian economics”

